Uniform Two-Part Tariff Pricing for Heterogeneous Consumers

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【Abstract】

Several forms of nonlinear pricing schedules exist across many industries. Although the fully optimal contract is theoretically very appealing, this pricing strategy remains difficult to apply in practice. Complex pricing schedule may not be profitable as predicted by theory. A simple two-part tariff is a reasonable marketing strategy for the firm. The paper examines the pricing behavior of a monopolist when his strategy space is restricted to two-part tariffs. In a general setting, a simple necessary condition for the optimal two-part tariff is derived. Based on the result, I derive analytical expressions of the optimal two-part tariff and the maximized expected profit. I examine how the degree of consumer heterogeneity affects the firm's pricing strategy and profitability if consumers' demand curves are linear and parallel.

【Keywords】 two-part tariff, heterogeneous consumers, asymmetric information

【JEL Classification Numbers】 D21, L12

Contents

1. Introduction

Nonlinear pricing has received considerable attention in the economic literature. The existing literature analyzing price discrimination generally suggests that the firm should employ a menu (a continuum) of two-part tariffs. Relying on the revelation principle, the menu of tariff options offered by the firm is described as a direct revelation mechanism in which the consumer reports his private information about taste or valuation. There is no doubt that the analysis of complex price discrimination strategies has shed light on several issues in nonlinear pricing, however the implementation of fully optimal contracts is too costly and prohibited in practice. Complex pricing schedule may not be profitable as predicted by theory. The paper examines the pricing behavior of a monopolist when his strategy space is restricted to two-part tariffs.

The existing literature on the second-degree price discrimination has focused on the fully optimal nonlinear pricing schedule, which is nowhere linear. For instance, Maskin and Riley (1984), Roberts (1979), and Wilson (1997) have concentrated on the nature of fully optimal pricing schemes. Notice that the fully optimal price schedule can be regarded as the envelope of a menu of infinitely many two-part tariffs in the standard model.

When the number of two-part tariffs is finite, the price schedule is referred to as a multiblock tariff. A one-block tariff is simply refereed to as a linear pricing strategy or a two-part tariff. It is obvious that the firm's profit will increase with the number of blocks in a multiblock tariff if there are many segments of consumers with different demand curves.[1] It would be valuable to understand how to determine the optimal two-part tariff.

When the firm is faced heterogeneous consumer tastes, but he is subject to uniform treatment of all consumers, how should he determine an optimal two-part tariff? The classical example of a two-part tariff is pricing an amusement park in which a customer must pay one price to enter the park, and further fees for each of the rides. More recently, Schmalensee (2015, p.19) addresses the "razor-and-blades" pricing strategy. A firm will set a low price for a basic product (a razor), and to earn all or most of its profits from sales of a complementary consumable product (blades). The classical exposition of two-part tariffs in a profit maximization setting is Oi (1971). He assumes a finite set of possible buyers with different demand curves and zero cost of the basic product, and mainly discusses whether it is always optimal to set the price of the consumable product above its marginal cost, rather than the impact of tariff choice on firm profitability. A more extended treatment may be found in Ng and Weisser (1974). Littlechild (1975) focuses on the use of two-part tariffs by welfare maximizing monopolist. Schmalensee (1981) extends the model to permit positive and constant unit costs for both basic and consumable products together with a continuum of consumers. On the contrary, the aim of the paper is to provide a new perspective regarding the optimal two-part tariff schedule. In a general setting, a simple necessary condition for the optimal two-part tariff is derived.

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1 See Lim and Ho (2007), Miravete (2004), and Murphy (1977) for the number of blocks question.

The necessary condition for the optimal two-part tariff derived in the paper is interpretable economically, however, it remains unclear how the degree of consumer heterogeneity does affect the firm's pricing strategy and profitability. To answer the question, I assume that consumers' demand curves are linear and parallel, and I restrict attention to a certain class of distribution function including the uniform distribution. The assumption of the uniform distribution or the assumption that there are the same numbers of consumers of each type is common in the literature. However, in the first place, a research question is that how firms use nonlinear pricing to benefit for the heterogeneity in consumer demand.[2] Explicit modeling of the firm's profit maximization problem benefits. I solve for not only the optimal two-part tariff schedule but also the analytical expression of the maximized expected profit for the firm implied by heterogeneity in consumer's valuation. Furthermore, I demonstrate how the firm takes account into consumer heterogeneity using the explicit solutions. In the aspect of quantitative analysis of two-part tariffs, the seminar paper by Leland and Meyer (1976) and the subsequent analysis by Mitchell (1978) are closely related to my work. To the best of my knowledge, they have not derived analytical expressions.

The rest of the paper is organized as follows: In the next section, I describe a model. Following this, I discuss the structure of the optimal two-part tariff schedule. I derive a necessary condition for the optimal two-part tariff schedule. In Section 4, based on the result derived in the previous section, I derive analytical expressions of the optimal two-part tariff and the maximized expected profit, and show how the degree of consumer heterogeneity affects the firm's pricing strategy and profitability if consumers' demand curves are linear and parallel. Section 5 concludes the paper.

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[2] See Lambrecht et al (2012) for this kind of perspective.

2. The Model

The paper studies the optimal two-part tariff by a monopolist facing heterogeneous consumer tastes. I consider a static model of a firm selling a single product that may be purchased in any quantity of units denoted by x. The firm produces the product at a constant marginal cost denoted by $c \geqslant 0$. There are no fixed costs. Thus, the total cost function of the firm is given by $C(x) = cx$.[3]

I assume that a continuum of potential buyers distributed according to some characteristic, with the property that demand increases with this characteristic. As mentioned in Schmalensee (1981, p.448), there is no need to make that characteristic explicit in general. Such characteristic is usually referred to as type. A consumer's type is identified by a single parameter θ. For analytical convenience, the index θ will be treated as a continuous variable. More precisely, the firm sells the product to a continuum of consumers who are distributed on the interval $[\theta_{\min}, \theta_{\max}]$. The difference between the two boundaries is given by $\Delta = \theta_{\max} - \theta_{\min}$, which is assumed to be $\Delta = 1$, as in Goldman et al (1984) and Oren et al (1983). Consumers have private information about θ that is a degree of heterogeneity in consumers' tastes for the product. The firm does not observe θ, but knows the distribution of θ represented by a distribution function $F(\theta)$ defined on the interval $[\theta_{\min}, \theta_{\max}]$. It is not necessarily distributed uniformly. In Section 4, I shall pay attention to a specified class of distribution functions including the uniform distribution to conduct comparative statics exercises. I assume that the distribution has positive density. Let $f(\theta)$ be the density

[3] The literature has focused on the case where marginal and average costs of production are constant with the exception of Monteiro and Page (1996) and Thomas (2001). They consider a more general cost functions depend on the aggregate supply.

of types. Each type θ may represent many consumers. Throughout the paper, I assume that the inverse hazard rate $(1-F(\theta))/f(\theta)$ is invertible.

The firm will specify a price schedule or an outlay function that depends on usage. A price schedule can be represented as a function $t(x)$ which is the charge for consuming the x units of the product. The paper restricts the firm's strategy. The firm is assumed to charge a two-part tariff $\{p,q\}$ to consumers, where p is the unit price for the consumable product and q is the membership fee or the price for the basic product. As Oi (1971, p.78) mentioned, the interpretation is that a two-part tariff is one in which the consumer must pay a lump sum fee q for the right to buy a product, and then a variable fee px which depends linearly on usage x. Precisely, a two-part tariff is defined by

$$t(x) = \begin{cases} px + q & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

In the paper, I assume that both components p and q are strictly positive. When the firm is allowed to use a fully optimal contract, it is well-known that it is optimal for the firm to post a concave price schedule in this framework (Maskin and Riley, 1984, Propisition 6). A two-part tariff is regarded as a simple approximation of such fully optimal contract.

For each quantity x, there is a maximum willingness to pay for type θ, denoted by $W(x,\theta)$. The marginal willingness to pay for the xth unit is given by $w(x,\theta) = \partial W(x,\theta)/\partial x$. The marginal willingness to pay function $w(x,\theta)$ can be interpreted as the inverse demand function of type θ. As is common in the literature on price discrimination, income effects are assumed negligible. Ignoring income effects, consumer's preferences are represented as the quasi-linear form:

$$\int_0^x w(\tilde{x},\theta)d\tilde{x} - t(x) \tag{1}$$

for each $x \geqslant 0$. The inverse demand function is assumed to be strictly decreasing and (weakly) convex in x, and strictly increasing in θ. These properties are summarized in Eq.(2).

$$w_x(x,\theta) < 0, \quad w_{xx}(x,\theta) \leqslant 0, \quad \text{and} \quad w_\theta(x,\theta) > 0. \tag{2}$$

Notice that demand curves are downward-sloping because of $w_x(x,\theta) < 0$. Also, $w_\theta(x,\theta) > 0$ implies that demand curves are ordered by θ. In other words, demand curves corresponding to different types never cross. This is the standard assumption in the literature, as Goldman et al (1984), Schmalensee (1981), and others, have assumed.

Each buyer has an optimal consumption level $x(p,\theta)$ that is obtained by maximizing his net utility. By the quasi-linearity of his preferences given by Eq.(1), his demand function depends on p and θ;

$$x(p,\theta) \in \text{argmax}\left[\int_0^x w(\tilde{x},\theta)d\tilde{x} - px \mid x \geqslant 0\right].$$

The consumer surplus is a crucial concept for the analysis in the paper. The consumer surplus when the consumer in excess of the membership fee is defined as usual:

$$CS(p,\theta) = \int_p^\infty x(\tilde{p},\theta)d\tilde{p}.$$

Equivalently,

$$CS(p,\theta) = \int_0^{x(p,\theta)} w(\tilde{x},\theta)d\tilde{x} - px(p,\theta). \tag{3}$$

I assume that the consumer of type θ will buy $x(p,\theta)$ units of the consumable good if and only if the basic good is purchased. That is, a buyer of type θ makes a purchase decision if and only if his consumer surplus is at least as high as the membership fee at his optimal consumption level, that is, $CS(p,\theta) \geqslant q$.

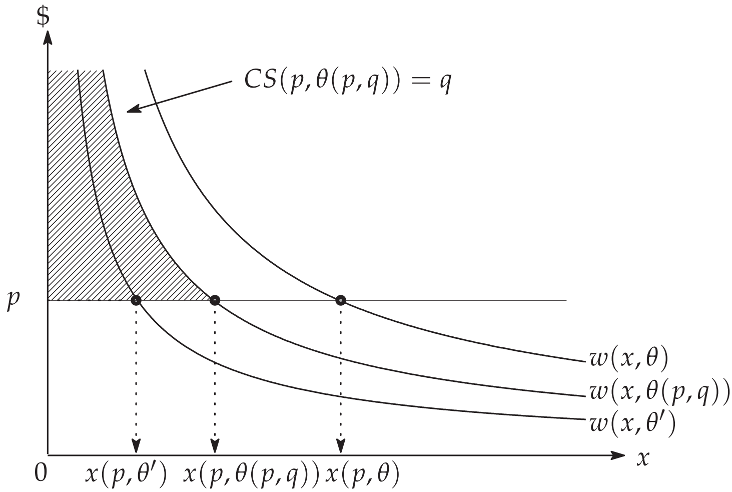

Figure 1 Demand Pattern under a two-part tariff: $\theta' < \theta(p,q) < \theta$

There must be the smallest type $\theta(p,q)$ of consumer with positive consumption under a two-part tariff $\{p,q\}$ as long as the condition $w_\theta(x,\theta) > 0$ is satisfied. Such monotonicity of $w(x,\theta)$ in θ ensures that the marginal type $\theta(p,q)$ defined implicitly by $CS(p,\theta(p,q)) = q$ is unique under a two-part tariff $\{p,q\}$. It is shown that the assumption $w_\theta(x,\theta) > 0$ implies that the consumer surplus $CS(p,\theta)$ is strictly increasing in θ, and thus all consumers with $\theta > \theta(p,q)$ purchase the product.

Restricting to a two-part tariff pricing, the consumer type $\theta(p,q)$ is uniquely determined (if it exists) by the condition $CS(p,\theta(p,q)) = q$, as shown in Figure 1. Since the unit price is constant, it follows that the marginal tariff line at p must intersects each demand curve $w(x,\theta)$ just once from below. Such intersection determines the optimal consumption level $x(p,\theta)$ as a solution to the utility maximization condition $w(x,\theta) = p$.

By definition of the marginal type $\theta(p,q)$, he consumes $x(p,\theta(p,q))$ and earns positive surplus which is equal to q. Any type $\theta' < \theta(p,q)$ is excluded from the market because he earns surplus which is strictly lower than q by consuming the best quantity $x(p,\theta')$ for himself. It is obvious that any type $\theta > \theta(p,q)$ stays in the market and consumes $x(p,\theta)$ because the resulting surplus is strictly higher than q for sure. Anyway, the condition $CS(p,\theta(p,q)) = q$ determines the lowest type $\theta(p,q)$ and the lowest quantity demanded $x(p,\theta(p,q))$ in the market. The argument so far suggests that the subscriber set is of the form $[\theta(p,q), \theta_{\max}]$, where $\theta(p,q)$ is the marginal type whose consumption surplus is exactly equal to the membership fee, that is, $CS(p,\theta(p,q)) = q$.

The partial derivatives of the consumer surplus provide useful insight. Firstly, the quantity demanded is derived by differentiating the consumer surplus with respect to price, referred to as *Roy's identity*: $CS_p(p,\theta) = \partial CS(p,\theta)/\partial p = -x(p,\theta)$, and thus, the quantity demanded is calculated as $x(p,\theta) = -CS_p(p,\theta)$. Secondly, the reason for that the subscriber set is a closed interval is based on the strict monotonicity of the consumer surplus in θ: $CS_\theta(p,\theta) > 0$.[4]

To end this section, I shall confirm the monotonicity of the quantity demanded $x(p,\theta)$ in θ, as depicted in Figure 1. Define the implicit function by $\varphi(x,\theta,p) = w(x,\theta) - p$. By definition of the quantity demanded $x(p,\theta)$, it satisfies $\varphi(x(p,\theta),p,\theta) = 0$. By the implicit function theorem, the partial derivative of $x(p,\theta)$ with respect to type θ is the following:

$$x_\theta(p,\theta) = -\frac{\partial\varphi(x(p,\theta),p,\theta)/\partial\theta}{\partial\varphi(x(p,\theta),p,\theta)/\partial x} = -\frac{w_\theta(x(p,\theta),\theta)}{w_x(x(p,\theta),\theta)} > 0. \tag{4}$$

[4] Differentiating the consumer surplus in Eq.(3) with respect to the unit price p to get $CS_p(p,\theta) = w(x(p,\theta),\theta)x_p(p,\theta) - x(p,\theta) - px_p(p,\theta) = [w(x(p,\theta),\theta) - p]x_p(p,\theta) - x(p,\theta) = -x(p,\theta)$. Next, the partial derivative of the consumer surplus with respect to type θ is written as $CS_\theta(p,\theta) = w(x(p,\theta),\theta)x_\theta(p,\theta) + \int_0^{x(p,\theta)} w_\theta(\tilde{x},\theta)d\tilde{x} - px_\theta(p,\theta) = [w(x(p,\theta),\theta) - p]x_\theta(p,\theta) + \int_0^{x(p,\theta)} w_\theta(\tilde{x},\theta)d\tilde{x} = \int_0^{x(p,\theta)} w_\theta(\tilde{x},\theta)d\tilde{x} > 0$.

This condition states that the quantity demanded is ordered by type θ. Similary, the partial derivative of $x(p, \theta)$ with respect to the unit price p is obtained:

$$x_p(p, \theta) = -\frac{\partial \varphi(x(p, \theta), p, \theta)/\partial p}{\partial \varphi(x(p, \theta), p, \theta)/\partial x} = -\frac{-1}{w_x(x(p, \theta), \theta)} < 0. \tag{5}$$

The monotonicity of the demand function in the unit price p in Eq.(5) will be used in the discussion in Section 4.

3. A Characterization for the Optimal Two-Part Tariff

The purpose of this section is twofold. Firstly, I shall formulate the objective function to be maximized by means of the demand of the average admitted consumers. Secondly, I shall derive a necessary condition for the optimal two-part tariff schedule. It is shown that the optimal two-part tariff must be set to equate two types of marginal rates of substitution that are defined for the profit contribution by the average admitted consumers and the marginal type, respectively. The arguments here are independent of the class of demand curves and distribution functions.

The analysis below mainly considers situations in which some but not all consumers buy the basic good, so that $\theta_{\min} < \theta(p, q) < \theta_{\max}$, as in Schmalensee (1981). In Section 4, when demand curves are parallel, it is shown that the polar case such that $\theta(p, q) = \theta_{\max}$ never occurs, and the second inequality $\theta(p, q) < \theta_{\max}$ holds indeed. On the other hand, the first inequality $\theta_{\min} < \theta(p, q)$ follows from a reasonable restriction to the set of parameters (see Assumption 1 below).

The aggregate demand of the admitted consumers is then

$$X(p,q) = \int_{\theta(p,q)}^{\theta\max} x(p,\theta) f(\theta) d\theta.$$

I can write the demand of the average admitted consumers as

$$Y(p,q) = X(p,q)/(1 - F(\theta(p,q))).$$

In terms of the demand of the average admitted consumers, the profit contribution by the average admitted consumers is written as

$$\tilde{\pi}(p,q) = (p-c)Y(p,q) + q.$$

The objective to be maximized is the following.

$$\begin{aligned}
\mathbb{E}\pi(p,q) &= \int_{\theta(p,q)}^{\theta\max} [t(x(p,\theta)) - C(x(p,\theta))] f(\theta) d\theta \\
&= \int_{\theta(p,q)}^{\theta\max} [\{px(p,\theta) + q\} - cx(p,\theta)] f(\theta) d\theta \\
&= (p-c) \int_{\theta(p,q)}^{\theta\max} x(p,\theta) f(\theta) d\theta + (1 - F(\theta(p,q)))q \\
&= (p-c)(1 - F(\theta(p,q))) \left(\frac{X(p,q)}{1 - F(\theta(p,q))} \right) + (1 - F(\theta(p,q)))q \\
&= [1 - F(\theta(p,q))][(p-c)Y(p,q) + q] \\
&= [1 - F(\theta(p,q))]\tilde{\pi}(p,q). \qquad (6)
\end{aligned}$$

Therefore, the objective function of the firm is written as the product of the proportion of the admitted consumers and the profit contribution by the average admitted consumers.

The literature on price discrimination, several papers have investigated the inverse elasticity rules. When there is a distribution of types as in the paper and the firm's strategy space is restricted to the class of two-part tariffs, Varian (1989, p.607) has derived the inverse elasticity rule based on the first-order condition with respect to the unit price only. In this sense, the inverse elasticity rule is a partial characterization of conditions for profit-maximizing behavior of the firm. On the contrary, the aim of the paper is to employ all first-order conditions. Consider the objective function given in Eq.(6). The first-order conditions are the following:

$$\begin{aligned}
&\frac{\partial \mathbb{E}\pi(p,q)}{\partial p} \\
&= -f(\theta(p,q))\theta_p(p,q)\tilde{\pi}(p,q) + (1 - F(\theta(p,q)))\tilde{\pi}_p(p,q) \\
&= f(\theta(p,q))\left\{\tilde{\pi}_p(p,q)\frac{1 - F(\theta(p,q))}{f(\theta(p,q))} - \tilde{\pi}(p,q)\theta_p(p,q)\right\} = 0,
\end{aligned}$$

and

$$\begin{aligned}
&\frac{\partial \mathbb{E}\pi(p,q)}{\partial q} \\
&= -f(\theta(p,q))\theta_q(p,q)\tilde{\pi}(p,q) + (1 - F(\theta(p,q)))\tilde{\pi}_q(p,q) \\
&= f(\theta(p,q))\left\{\tilde{\pi}_q(p,q)\frac{1 - F(\theta(p,q))}{f(\theta(p,q))} - \tilde{\pi}(p,q)\theta_q(p,q)\right\} = 0.
\end{aligned}$$

Since the distribution has positive density, it follows that the optimal two-part tariff is regarded as a solution to the following system of symmetric equations:

$$\begin{cases}
\phi_1(p,q) = \tilde{\pi}_p(p,q)\dfrac{1 - F(\theta(p,q))}{f(\theta(p,q))} - \tilde{\pi}(p,q)\theta_p(p,q) = 0, \\
\phi_2(p,q) = \tilde{\pi}_q(p,q)\dfrac{1 - F(\theta(p,q))}{f(\theta(p,q))} - \tilde{\pi}(p,q)\theta_q(p,q) = 0.
\end{cases} \tag{7}$$

Although the two components of two-part tariffs may play dissimilar roles, the first-order conditions are symmetric. Notice that $\phi_1(p,q) = 0$ yields that

$$\tilde{\pi}_p(p,q)\theta_p^{-1}(p,q) = \tilde{\pi}(p,q)\left(\frac{1-F(\theta(p,q))}{f(\theta(p,q))}\right)^{-1}. \tag{8}$$

Similarly, $\phi_2(p,q) = 0$ yields that

$$\tilde{\pi}_q(p,q)\theta_q^{-1}(p,q) = \tilde{\pi}(p,q)\left(\frac{1-F(\theta(p,q))}{f(\theta(p,q))}\right)^{-1}. \tag{9}$$

The system of equations in Eq.(7) provides an interesting economic insight. Rearranging the two equations, Eq.(8) and Eq.(9), I have the following proposition.

Proposition 1 (necessary condition for the optimal price structure)**.** The optimal two-part tariff is determined so that the marginal profit contribution by the average admitted consumers per the marginal effect on the marginal type is equalized:

$$\frac{\tilde{\pi}_p(p,q)}{\theta_p(p,q)} = \frac{\tilde{\pi}_q(p,q)}{\theta_q(p,q)} \quad \text{or} \quad \frac{\tilde{\pi}_p(p,q)}{\tilde{\pi}_q(p,q)} = \frac{\theta_p(p,q)}{\theta_q(p,q)}. \tag{10}$$

Several works such as Oi (1971), Schmalensee (1981), and Varian (1989) have examined the properties of the optimal two-part tariff and studied whether a monopolist should set the unit price of the consumable product above its marginal cost. To answer this question, they have employed only the first-order condition with respect to the unit price, which is interpreted as a partial characterization of the optimal two-part tariff. On the other hand, the above proposition describes the condition taking into account the entire system of the first-order conditions. In this sense, the paper obtains a full characterization of the optimal two-part tariff.

4. Optimal Price Structure: Linear Demand Case

I shall illustrate some features of the profit-maximizing two-part tariff strategy. In this section, I restrict attention to the marginal willingness to pay function of the form $w(x,\theta)=\theta-x$ for each type θ.[5] All the demand curves are parallel. The assumption that linear demand curves are exactly parallel is familiar in the literature. The seminar paper by Leland and Meyer (1976) consider the same demand curve and subsequent analysis by Schmalensee (2015) as well.

In this case, the first-order condition for the utility maximization is written as $w(x,\theta)=\theta-x=p$. The corresponding demand function is given by $x(p,\theta)=\theta-p$, provided that his consumer surplus is at least as high as the membership fee. The consumer surplus is calculated as the following:

$$
\begin{aligned}
CS(p,\theta) &= \int_0^{x(p,\theta)} w(\tilde{x},\theta)d\tilde{x} - px(p,\theta) \\
&= \int_0^{\theta-p} (\theta-\tilde{x})d\tilde{x} - p(\theta-p) \\
&= \theta(\theta-p) - 0.5(\theta-p)^2 - p(\theta-p) \\
&= 0.5(\theta-p)^2.
\end{aligned}
$$

[5] I may use a more general marginal willingness to pay function defined by $w(x,\theta)=a+\theta-bx$. The corresponding demand function is written as $x(p,\theta)=(a+\theta-p)/b$ from the first-order condition $w(x,\theta)=p$. But there is no significant difference in results.

Recall that any type $\theta \geqslant \theta(p,q)$ will be active in the market. The marginal type $\theta(p,q)$ is obtained by solving the equation $CS(p,\theta) = q$ for θ. The equation $0.5(\theta - p)^2 = q$ has two roots: $\theta = p \pm \sqrt{2q}$. I conclude that the marginal type is given by $\theta(p,q) = p + \sqrt{2q}$ because only the bigger of the two roots is consistent with the fact that the quantity demanded by the marginal type is positive: $x(p,\theta(p,q)) = \theta(p,q) - p > 0$. Therefore, the demand function under the two-part tariff $\{p,q\}$ is summarized by Eq.(11):

$$x(p,\theta) = \begin{cases} \theta - p & \text{if } \theta \geqslant \theta(p,q) = p + \sqrt{2q}, \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

To calculate the profit-maximizing price schedule, I need to specify a class of distribution function. Most of the works found in the literature have assumed that the index θ is uniformly distributed. This paper, on the other hand, is concerned with the profit-maximizing price structure under non-uniform distribution of buyer types. Consider the distribution function defined in Eq.(12):

$$F(\theta) = 1 - \left(\frac{\theta_{\max} - \theta}{\Delta}\right)^{\frac{1}{\eta}} = 1 - \left(1 - \frac{\theta - \theta_{\min}}{\Delta}\right)^{\frac{1}{\eta}}, \tag{12}$$

where $\Delta = \theta_{\max} - \theta_{\min}$ and $\eta \in (0,\infty)$.[6] Notice that the distribution coincides with the uniform when $\eta = 1$. Roughly speaking, the distribution function has a value near the lower bound $\theta_{\min}$ when $\eta \in (0,1)$, whereas the distribution has a value near the upper bound $\theta_{\max}$ when $\eta \in (1,\infty)$. The distribution becomes more favorable to the firm when η increases, because a higher value of η means that there is a larger proportion of highly motivated

[6] This kind of a distribution is referred to as a Burr type XII distribution in Miravete (2004). He employed this type of distribution function to analyze some properties of multiblock tariffs.

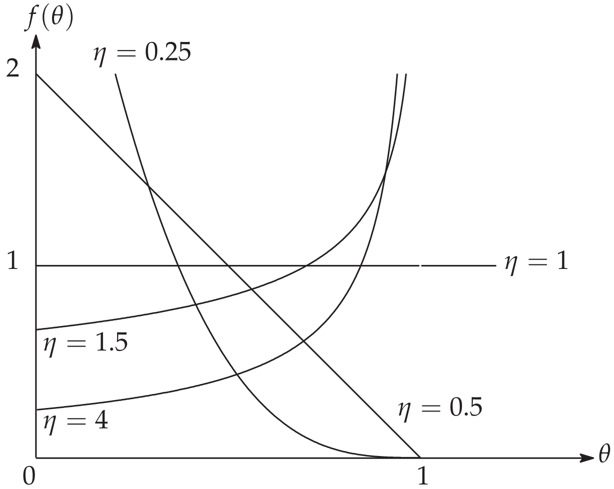

Figure 2　Illustration of the density function ($\theta_{\max} = 1, \theta_{\min} = 0$)

consumers. In fact, the parameter η is interpreted as a *first-order stochastic dominance shift parameter.*[7] The distribution function is flexible, and can accommodate J- and inverted-J-shapes.[8] Figure 2 illustrates the shape of the density function for some alternative values of η ranging between 0.25 and 4.

This distribution allows for closed solutions of optimal contracts discussed below. For this class of distribution in Eq.(12), the inverse hazard rate is linear in private information. This is calculated explicitly as shown in Eq.(13). The monotonicity of the inverse hazard rate is automatically

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7 $dF(\theta;\eta)/d\eta = (\theta_{\max} - \theta)^{\frac{1}{\eta}} \ln\left((\theta_{\max} - \theta)/\Delta\right)/\eta^2 \Delta^{\frac{1}{\eta}} \leqslant 0$ with equality only if $\theta = \theta_{\min}$.

8 A couple of previous works consider a deviation from the uniform distribution when adverse selection is a potential concern. For example, Reichelstein (1992) and Chu and Sappington (2007) in the procurement problem. Also, Rao (1990) in the context of salesforce compensation.

satisfied.

$$\frac{1-F(\theta)}{f(\theta)} = \eta(\theta_{\max} - \theta). \tag{13}$$

Some useful property of the above family of distributions are summarized by the following lemma. The proof is omitted.

Lemma 1 (expectation)**.** When $\tilde{\theta} = \theta_{\max} - \varepsilon$ for some $\varepsilon \geqslant 0$,

$$\int_{\theta_{\min}}^{\tilde{\theta}} \theta f(\theta) d\theta = \mathbb{E}(\theta) - (1 - F(\tilde{\theta})) \left(\mathbb{E}(\theta) + \frac{\Delta - \varepsilon}{\eta + 1} \right), \tag{14}$$

where the expectation over the entire type space is given by

$$\mathbb{E}(\theta) = \frac{\eta\theta_{\max} + \theta_{\min}}{\eta + 1}.$$

By Lemma 1, the average type over the interval $[\hat{\theta}, \theta_{\max}]$ can be calculated as

$$\mathbb{E}[\theta \mid \theta \geqslant \hat{\theta}] = \frac{\int_{\hat{\theta}}^{\theta_{\max}} \theta f(\theta) d\theta}{1 - F(\hat{\theta})} = \mathbb{E}(\theta) + \frac{\hat{\theta} - \theta_{\min}}{\eta + 1} = \frac{\eta\theta_{\max} + \hat{\theta}}{\eta + 1},$$

where the second equality holds because $\Delta - \varepsilon = (\theta_{\max} - \theta_{\min}) - (\theta_{\max} - \hat{\theta}) = \hat{\theta} - \theta_{\min}$ in Eq.(14). I conclude that

$$\int_{\hat{\theta}}^{\theta_{\max}} \theta f(\theta) d\theta = (1 - F(\hat{\theta})) \left(\frac{\eta\theta_{\max} + \hat{\theta}}{\eta + 1} \right). \tag{15}$$

Before starting the analysis in this framework, I need to impose a restriction to the set of parameters to get an economically meaningful solution. I would like to ensure that it is socially desirable to serve all the consumers under complete information. When I consider the situation in which all consumers are the same type θ or there is no information asymmetry between the firm and the consumers, the optimal price schedule is obtained by solving the following profit maximization problem:

$$\max\,(p-c)x(p,\theta)+q \quad \text{subject to } CS(p,\theta)\geqslant q.$$

The firm charges an membership fee q so that the consumer will choose to stay in the market, that is, $CS(p,\theta)\geqslant q$. It is obvious that it is optimal for the firm to set maximum entrance fee to extract all the consumer's surplus, and thus the constraint must be binding. As in Varian (1989, pp.604–605), incorporating the binding constraint into the objective function and differentiating with respect to the unit price p yields $0 = x(p,\theta)+(p-c)x_p(p,\theta)+CS_p(p,\theta) = x(p,\theta)+(p-c)x_p(p,\theta)-x(p,\theta) = (p-c)x_p(p,\theta)$, where the last equality is due to Roy's identity. Therefore, for each type θ, the corresponding first order condition $(p-c)x_p(p,\theta) = 0$ yields the standard marginal cost pricing policy, $p = c$ since $x_p(p,\theta) < 0$ by Eq.(5). That is, the optimal policy is to engage in first-degree price discrimination. In this linear demand model, the quantity demanded is given by $x(c,\theta) = \theta - c$, and thus it suffices to assume that $\theta_{\min} > c$ if all the consumers must be served under complete information.

Assumption 1 (no exclusion under complete information)**.** The marginal cost must be sufficiently low so that $\theta_{\min} > c$.

To apply Proposition 1 to derive the optimal two-part tariff explicitly, I need to calculate the partial derivatives of the profit contribution by the average admitted consumers and the marginal type. Since the partial derivatives of the profit contribution by the average admitted consumers involve the expressions for $Y_p(p,q)$ and $Y_q(p,q)$, I first calculate the demand of the average admitted consumers $Y(p,q)$. Under the specified demand curve $x(p,\theta) = \theta - p$, the aggregate demand of the admitted consumers is given by

$$\begin{aligned}
X(p,q) &= \int_{\theta(p,q)}^{\theta_{\max}} x(p,\theta) f(\theta) d\theta \\
&= \int_{\theta(p,q)}^{\theta_{\max}} (\theta - p) f(\theta) d\theta \\
&= \int_{\theta(p,q)}^{\theta_{\max}} \theta f(\theta) d\theta - (1 - F(\theta(p,q))p \\
&= (1 - F(\theta(p,q))) \left(\frac{\eta\theta_{\max} + \theta(p,q)}{\eta + 1} \right) - (1 - F(\theta(p,q)))p \\
&= (1 - F(\theta(p,q)) \left(\frac{\eta\theta_{\max} + \theta(p,q)}{\eta + 1} - p \right)
\end{aligned}$$

because Eq.(15) evaluated at $\hat{\theta} = \theta(p,q)$ is given by

$$\int_{\theta(p,q)}^{\theta_{\max}} \theta f(\theta) d\theta = [1 - F(\theta(p,q))] \left(\frac{\eta\theta_{\max} + \theta(p,q)}{\eta + 1} \right).$$

By definition of the demand of the average admitted consumers, substituting $\theta(p,q) = p + \sqrt{2q}$ into the expression inside the round brackets yields

$$\begin{aligned}
Y(p,q) &= \frac{\eta\theta_{\max} + \theta(p,q)}{\eta + 1} - p \\
&= \frac{\eta\theta_{\max} + p + \sqrt{2q} - (\eta + 1)p}{\eta + 1} \\
&= \frac{\eta(\theta_{\max} - p) + \sqrt{2q}}{\eta + 1}.
\end{aligned}$$

Consequently, the expressions related to aggregate demands are of the following forms:

$$X(p,q) = (1 - F(\theta(p,q))Y(p,q),$$
$$\text{where} \quad Y(p,q) = \frac{\eta(\theta_{\max} - p) + \sqrt{2q}}{\eta + 1}. \tag{16}$$

I am ready to get the partial derivatives of the demand and the profit contribution of the average admitted consumers, respectively, under the linear demand structure. From Eq.(16), the partial derivatives of the demand of the average admitted consumers are the following:

$$Y_p(p,q) = -\frac{\eta}{\eta+1} \quad \text{and} \quad Y_q(p,q) = \frac{1}{(\eta+1)\sqrt{2q}}.$$

It then follows that

$$\begin{aligned}
\tilde{\pi}_p(p,q) &= Y(p,q) + (p-c)Y_p(p,q) \\
&= \frac{\eta(\theta_{\max} - p) + \sqrt{2q}}{\eta+1} - \frac{\eta(p-c)}{\eta+1} \\
&= \frac{\eta(\theta_{\max} - 2p + c) + \sqrt{2q}}{\eta+1},
\end{aligned}$$

and

$$\begin{aligned}
\tilde{\pi}_q(p,q) &= (p-c)Y_q(p,q) + 1 \\
&= \frac{p-c}{(\eta+1)\sqrt{2q}} + 1 \\
&= \frac{p - c + (\eta+1)\sqrt{2q}}{(\eta+1)\sqrt{2q}}.
\end{aligned}$$

Furthermore, the partial derivatives of the marginal type are the following: $\theta_p(p,q) = \partial\theta(p,q)/\partial p = 1$ and $\theta_q(p,q) = \partial\theta(p,q)/\partial q = 1/\sqrt{2q}$. I would like to solve the equation $\tilde{\pi}_p(p,q)\theta_p^{-1}(p,q) = \tilde{\pi}_q(p,q)\theta_q^{-1}(p,q)$ for the term $\sqrt{2q}$. The equivalence between the two marginal rates of substitution in Eq.(10) becomes:

$$\frac{\eta(\theta_{\max} - 2p + c) + \sqrt{2q}}{(p - c + (\eta+1)\sqrt{2q})/\sqrt{2q}} = \sqrt{2q}.$$

Dividing both sides by $\sqrt{2q}$ and rearranging the expression yields

$$\eta(\theta_{\max} - 2p + c) + \sqrt{2q} = p - c + (\eta+1)\sqrt{2q}.$$

Rearranging gives

$$\sqrt{2q} = \frac{\eta(\theta_{\max} - 2p + c) - (p - c)}{\eta} = \frac{\eta\theta_{\max} - (2\eta+1)p + (\eta+1)c}{\eta}, \quad (17)$$

and then

$$q = \frac{(\eta\theta_{\max} - (2\eta+1)p + (\eta+1)c)^2}{2\eta^2}. \quad (18)$$

Substituting Eq.(17) and Eq.(18) into $\phi_2(p,q) = 0$ to get the expression for the unit price p. Firstly, I need to simplify the expression for $\phi_2(p,q) = 0$ as a function of p and $\sqrt{2q}$:

$$\begin{aligned}
0 &= \phi_2(p,q) \\
&= \tilde{\pi}_q(p,q)\frac{1 - F(\theta(p,q))}{f(\theta(p,q))} - \tilde{\pi}(p,q)\theta_q(p,q) \\
&= \frac{\eta(p - c + (\eta+1)\sqrt{2q})(\theta_{\max} - p - \sqrt{2q})}{(\eta+1)\sqrt{2q}} \\
&\quad - \left(\frac{(p-c)(\eta(\theta_{\max} - p) + \sqrt{2q})}{\eta+1} + q\right)\left(\frac{1}{\sqrt{2q}}\right)
\end{aligned}$$

$$= \left(\frac{1}{(\eta+1)\sqrt{2q}}\right)\Big[\eta(p-c+(\eta+1)\sqrt{2q})(\theta_{\max}-p-\sqrt{2q})$$

$$-\Big((p-c)(\eta(\theta_{\max}-p)+\sqrt{2q})+(\eta+1)q\Big)\Big]. \tag{19}$$

Notice that the expression inside the square brackets on the right-hand side in Eq.(19) becomes:

$$\eta(p-c+(\eta+1)\sqrt{2q})(\theta_{\max}-p-\sqrt{2q})$$

$$-\Big((p-c)(\eta(\theta_{\max}-p)+\sqrt{2q})+(\eta+1)q\Big)$$

$$=\eta(p-c)(\theta_{\max}-p)+\eta(\eta+1)(\theta_{\max}-p)\sqrt{2q}$$

$$-\eta(p-c)\sqrt{2q}-2\eta(\eta+1)q-\eta(p-c)(\theta_{\max}-p)$$

$$-(p-c)\sqrt{2q}-(\eta+1)q$$

$$=\eta(\eta+1)(\theta_{\max}-p)\sqrt{2q}-(\eta+1)(p-c)\sqrt{2q}-(\eta+1)(2\eta+1)q$$

$$=(\eta+1)\sqrt{2q}\left(\eta(\theta_{\max}-p)-(p-c)-(2\eta+1)q/\sqrt{2q}\right).$$

Therefore, the first-order condition $\phi_2(p,q)=0$ is simplified to

$$0=\eta(\theta_{\max}-p)-(p-c)-\frac{(2\eta+1)q}{\sqrt{2q}}$$

$$=\eta\theta_{\max}-(\eta+1)p+c-\frac{(2\eta+1)q}{\sqrt{2q}}.$$

Since $\sqrt{2q}\phi_2(p,q) = 0$ holds, it follows that

$$(\eta\theta_{\max} - (\eta+1)p + c)\sqrt{2q} = (2\eta+1)q. \tag{20}$$

Substitute Eq.(17) and Eq.(18) into Eq.(20) and solve for the unit price p. It is shown that there are two possible solutions. Eq.(20) becomes:

$$\frac{(\eta\theta_{\max} - (\eta+1)p + c)(\eta\theta_{\max} - (2\eta+1)p + (\eta+1)c)}{\eta}$$

$$= \frac{(2\eta+1)(\eta\theta_{\max} - (2\eta+1)p + (\eta+1)c)^2}{2\eta^2}.$$

Multiplying $2\eta^2$ to both sides yields

$$\begin{aligned}
0 &= 2\eta(\eta\theta_{\max} - (\eta+1)p + c)(\eta\theta_{\max} - (2\eta+1)p + (\eta+1)c) \\
&\quad -(2\eta+1)(\eta\theta_{\max} - (2\eta+1)p + (\eta+1)c)^2 \\
&= (\eta\theta_{\max} - (2\eta+1)p + (\eta+1)c) \\
&\quad \times(2\eta(\eta\theta_{\max} - (\eta+1)p + c) - (2\eta+1)(\eta\theta_{\max} - (2\eta+1)p + (\eta+1)c)) \\
&= -(\eta\theta_{\max} - (2\eta+1)p + (\eta+1)c) \\
&\quad \times(\eta\theta_{\max} - (2\eta^2+2\eta+1)p + (2\eta^2+\eta+1)c)).
\end{aligned} \tag{21}$$

Then, as mentioned earlier, there are two possible solutions to Eq.(21):

$$p_1 = \frac{\eta\theta_{\max} + (\eta+1)c}{2\eta+1} \quad \text{and} \quad p_2 = \frac{\eta\theta_{\max} + (2\eta^2+\eta+1)c}{2\eta^2+2\eta+1}.$$

Denote the corresponding membership fees by q_1 and q_2, respectively. Notice that $\sqrt{2q_1} = 0$ for the candidate p_1, however, this implies that the marginal type will consume nothing, which contradicts the assumption that his consumption is strictly positive:

$$x(p_1, \theta(p_1, q_1)) = \theta(p_1, q_1) - p_1 = \sqrt{2q_1} = 0.$$

Thus, this case is ruled out. On the other hand, for the candidate p_2, the demand of the marginal consumer is positive indeed:

$$x(p_2, \theta(p_2, q_2)) = \theta(p_2, q_2) - p_2 = \sqrt{2q_2} = \frac{2\eta^2(\theta_{\max} - c)}{2\eta^2 + 2\eta + 1} > 0$$

because

$$\begin{aligned}
\sqrt{2q_2} &= \frac{\eta\theta_{\max} - (2\eta + 1)p_2 + (\eta + 1)c}{\eta} \\
&= \frac{\eta\theta_{\max} + (\eta + 1)c}{\eta} - \frac{(2\eta + 1)(\eta\theta_{\max} + (2\eta^2 + \eta + 1)c)}{\eta(2\eta^2 + 2\eta + 1)} \\
&= \frac{2\eta^3(\theta_{\max} - c)}{\eta(2\eta^2 + 2\eta + 1)} \\
&= \frac{2\eta^2(\theta_{\max} - c)}{2\eta^2 + 2\eta + 1}.
\end{aligned}$$

This yields the corresponding membership fee:

$$q_2 = \frac{2\eta^4(\theta_{\max} - c)^2}{(2\eta^2 + 2\eta + 1)^2}.$$

I have obtained the optimal two-part tariff strategy as a function of $(\theta_{\max}, c, \eta)$. I would like to emphasize the dependence of the pricing strategy on the first-order stochastic shift parameter η for comparative statics exercises.

Proposition 2 (optimal price schedule)**.** The optimal two-part tariff schedule consists of $\{p(\eta), q(\eta)\}$:

$$\begin{cases} p(\eta) = \dfrac{\eta\theta_{\max} + (2\eta^2 + \eta + 1)c}{2\eta^2 + 2\eta + 1}, \\ q(\eta) = \dfrac{2\eta^4(\theta_{\max} - c)^2}{(2\eta^2 + 2\eta + 1)^2}. \end{cases} \tag{22}$$

Here, the graph of the optimal unit price becomes humped in general, whereas the graph of the optimal membership fee is strictly increasing as η increases since $\theta_{\max} > c$ holds under Assumption 1:

$$\frac{dp(\eta)}{d\eta} = -\frac{(2\eta^2 - 1)(\theta_{\max} - c)}{(2\eta^2 + 2\eta + 1)^2},$$

$$\frac{dq(\eta)}{d\eta} = \frac{8\eta^3(\eta + 1)(\theta_{\max} - c)}{(2\eta^2 + 2\eta + 1)^3}.$$

Under the uniform distribution ($\eta = 1$), the sign of the derivative of the unit price is strictly positive for sure, however, it would be negative for sufficiently small $\eta > 1/\sqrt{2} \approx 0.7071$.

Since the marginal cost of the admission is assumed to be zero, it is obvious that the price-cost margin for the basic good is strictly positive. Also, the price-cost margin for the variable commodity is strictly positive for the range of parameters satisfying Assumption 1:

$$p - c = \frac{\eta\theta_{\max} + (2\eta^2 + \eta + 1)c}{2\eta^2 + 2\eta + 1} - c = \frac{\eta(\theta_{\max} - c)}{2\eta^2 + 2\eta + 1} > 0.$$

Corollary 1 (mark-up)**.** The consumable good is sold above the marginal cost under the optimal two-part tariff.

I shall examine the effect of a change in η on the marginal type. I see that

$$
\begin{aligned}
\theta(\eta) &= \theta(p(\eta), q(\eta)) \\
&= p(\eta) + \sqrt{2q(\eta)} \\
&= \frac{\eta\theta_{\max} + (2\eta^2 + \eta + 1)c}{2\eta^2 + 2\eta + 1} + \frac{2\eta^2(\theta_{\max} - c)}{2\eta^2 + 2\eta + 1} \\
&= \frac{(\eta + 2\eta^2)\theta_{\max} + (2\eta^2 + \eta + 1 - 2\eta^2)c}{2\eta^2 + 2\eta + 1} \\
&= \frac{\eta(2\eta + 1)\theta_{\max} + (\eta + 1)c}{2\eta^2 + 2\eta + 1}.
\end{aligned}
$$

This implies that the marginal type is strictly increasing in η:

$$
\frac{d\theta(p(\eta), q(\eta))}{d\eta} = \frac{(2\eta^2 + 4\eta + 1)(\theta_{\max} - c)}{(2\eta^2 + 2\eta + 1)^2} > 0.
$$

Besides, under Assumption 1, the induced marginal type $\theta(p(\eta), q(\eta))$ is always lower than the upper boundary $\theta_{\max}$ because

$$
\begin{aligned}
\theta_{\max} - \theta(p(\eta), q(\eta)) &= \theta_{\max} - \frac{\eta(2\eta + 1)\theta_{\max} + (\eta + 1)c}{2\eta^2 + 2\eta + 1} \\
&= \frac{(2\eta^2 + 2\eta + 1)\theta_{\max} - \{\eta(2\eta + 1)\theta_{\max} + (\eta + 1)c\}}{2\eta^2 + 2\eta + 1} \\
&= \frac{(\eta + 1)(\theta_{\max} - c)}{2\eta^2 + 2\eta + 1} > 0.
\end{aligned}
$$

Notice the last equality holds because $\theta_{\max} - c > \theta_{\min} - c > 0$ under Assumption 1. Next, it is shown that the induced marginal type is higher than the lower boundary $\theta_{\min}$ only for sufficiently high values of the first-order stochastic shift parameter η:

$$
\begin{aligned}
\theta(p(\eta), q(\eta)) - \theta_{\min} &= \frac{\eta(2\eta+1)\theta_{\max} + (\eta+1)c}{2\eta^2 + 2\eta + 1} - \theta_{\min} \\
&= \frac{\eta(2\eta+1)\theta_{\max} + (\eta+1)c - (2\eta^2 + 2\eta + 1)\theta_{\min}}{2\eta^2 + 2\eta + 1} \\
&= \frac{\eta(2\eta+1)(\theta_{\max} - \theta_{\min}) - (\eta+1)(\theta_{\min} - c)}{2\eta^2 + 2\eta + 1} \\
&= \frac{2\eta^2 + \eta(1-w) - w}{2\eta^2 + 2\eta + 1}, \qquad (23)
\end{aligned}
$$

where $w = \theta_{\min} - c$. Under Assumption 1, this auxiliary value w is strictly positive.

To implement comparative statics exercises, the relevant set of parameters must be identified. When the boundaries of the domain of the distribution function, $\theta_{\min}$ and $\theta_{\max}$, and the marginal cost c are fixed, it is shown that the first-order stochastic dominance shift parameter η must be bounded from below. When the numerator in Eq.(23) is nonnegative, the marginal type should belong to the type space. It is obvious that the function $\phi(\eta; w)$ defined below is strictly convex in η, and it is minimized at $\eta = 0.25(1-w)$ for every $w = \theta_{\min} - c > 0$, where $\phi(\eta; w) = 2\eta^2 + (1-w)\eta - w$. The quadratic function $\phi(\eta; w)$ has two roots: $0.25(-1+w \pm \sqrt{w^2 + 6w + 1})$. It turns out that $\phi(\eta; w) > 0$ for $\eta > \eta(w)$, where $\eta(w) = 0.25(-1 + w + \sqrt{w^2 + 6w + 1})$, as shown in Figure 3. [9]

To end this section, I shall derive the relevant expressions induced by the optimal two-part tariff strategy. Recall that the expected profit is written

[9] Let $w > 0$. Since $\sqrt{w^2 + 6w + 1} - (-1 + w) > \sqrt{w^2 + 2w + 1} - (-1 + w) = w + 1 - (-1 + w) = 2 > 0$, it follows that the root $0.25(-1 + w - \sqrt{w^2 + 6w + 1})$ must be negative. On the other hand, notice that $\sqrt{w^2 + 6w + 1} - 1 + w > \sqrt{w^2 + 2w + 1} - 1 + w = w + 1 - 1 + w = 2w > 0$, which implies that the root $0.25(-1 + w + \sqrt{w^2 + 6w + 1})$ must be positive.

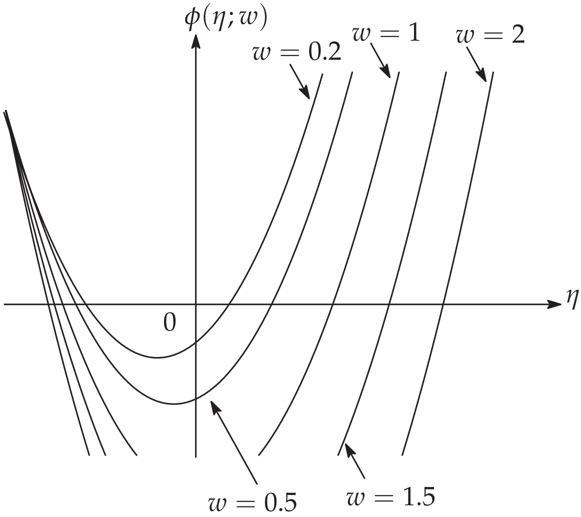

Figure 3　Lower bounds for the first-order stochastic shift paramter with several values of $w=\theta_{\min}-c$

as the product of two terms:

$$\mathbb{E}\pi(p,q)=(1-F(\theta(p,q)))[(p-c)Y(p,q)+q]. \tag{24}$$

Evaluating the first term on the right-hand side of Eq.(24) at the optimal two-part tariff $\{p(\eta),q(\eta)\}$, the volume of the admitted consumers is given by

$$\begin{aligned}1-F(\theta(p(\eta),q(\eta))) &= \left(\frac{\theta_{\max}-\theta(p(\eta),q(\eta))}{\Delta}\right)^{\frac{1}{\eta}}\\ &= \left(\frac{(2\eta^2+2\eta+1-\eta(2\eta+1))\theta_{\max}-(\eta+1)c}{(2\eta^2+2\eta+1)\Delta}\right)^{\frac{1}{\eta}}\\ &= \left(\frac{(\eta+1)(\theta_{\max}-c)}{(2\eta^2+2\eta+1)\Delta}\right)^{\frac{1}{\eta}}.\end{aligned}$$

Also, the second term on the right-hand side of Eq.(24), that is, the demand of the average admitted consumers, is written as

$$Y(\eta) = Y(p(\eta), q(\eta)) = \frac{\eta(2\eta+1)(\theta_{\max}-c)}{2\eta^2+2\eta+1}.$$

Then, the profit contribution by the average admitted consumers becomes:

$$\begin{aligned}
\tilde{\pi}(\eta) &= \tilde{\pi}(p(\eta), q(\eta)) \\
&= (p(\eta)-c)Y(\eta) + q(\eta) \\
&= \left(\frac{\eta(\theta_{\max}-c)}{2\eta^2+2\eta+1}\right)\left(\frac{\eta(2\eta+1)(\theta_{\max}-c)}{2\eta^2+2\eta+1}\right) + \frac{2\eta^4(\theta_{\max}-c)^2}{(2\eta^2+2\eta+1)^2} \\
&= \frac{(\eta^2(2\eta+1)+2\eta^4)(\theta_{\max}-c)^2}{(2\eta^2+2\eta+1)^2} \\
&= \frac{\eta^2(2\eta^2+2\eta+1)(\theta_{\max}-c)^2}{(2\eta^2+2\eta+1)^2} \\
&= \frac{\eta^2(\theta_{\max}-c)^2}{2\eta^2+2\eta+1}.
\end{aligned}$$

Thus, the maximized expected profit is written as a function of η:

$$\begin{aligned}
\mathbb{E}\pi(\eta) &= \mathbb{E}\pi(p(\eta), q(\eta)) \\
&= \left(\frac{(\eta+1)(\theta_{\max}-c)}{(2\eta^2+2\eta+1)\Delta}\right)^{\frac{1}{\eta}}\left(\frac{\eta^2(\theta_{\max}-c)^2}{2\eta^2+2\eta+1}\right) \\
&= \left(\frac{\eta^2(\theta_{\max}-c)}{\eta+1}\right)\left(\frac{(\eta+1)(\theta_{\max}-c)}{2\eta^2+2\eta+1}\right)^{\frac{\eta+1}{\eta}}.
\end{aligned}$$

Corollary 2 (summary)**.** Analytical expressions for profit-maximizing two-part tariff are summarized in Table 1:

Table 1　Profit maximizing pricing strategy

Unit price	$p(\eta) = \dfrac{\eta\theta_{\max} + (2\eta^2 + \eta + 1)c}{2\eta^2 + 2\eta + 1}$
Membership fee	$q(\eta) = \dfrac{2\eta^4(\theta_{\max} - c)^2}{(2\eta^2 + 2\eta + 1)^2}$
Marginal type	$\theta(\eta) = \dfrac{\eta(2\eta + 1)\theta_{\max} + (\eta + 1)c}{2\eta^2 + 2\eta + 1}$
Average demand	$Y(\eta) = \dfrac{\eta(2\eta + 1)(\theta_{\max} - c)}{2\eta^2 + 2\eta + 1}$
Average profit contribution	$\tilde{\pi}(\eta) = \dfrac{\eta^2(\theta_{\max} - c)^2}{2\eta^2 + 2\eta + 1}$
Maximized expected profit	$\mathbb{E}\pi(\eta) = \left(\dfrac{\eta^2(\theta_{\max} - c)}{\eta + 1}\right)\left(\dfrac{(\eta + 1)(\theta_{\max} - c)}{2\eta^2 + 2\eta + 1}\right)^{\frac{\eta+1}{\eta}}$

For numerical simulations, I chose the following two combinations of parameters appeared in Table 2. Recall that $\eta(w)$, where $w = \theta_{\min} - c$, is the lower bound on the value of the first-order stochastic shift parameter η so that the marginal type $\theta(\eta)$ belongs to the type space. The marginal cost c is positive in the first scenario, whereas it is zero in the second scenario. In other words, the latter case corresponds to a revenue-maximization problem.

Several properties of the optimal two-part tariff can be observed. All of expressions in Table 1, except the unit price for the consumable good, may increase when the distribution of the heterogeneity of potential buyers shifts to the right (i.e., when the first-order stochastic shift parameter η increases). The unit price will rise for a range of the parameter η, but it tends

Table 2 Relevant parameters for numerical simulations

$\theta_{\max}$	$\theta_{\min}$	c	$\eta(w)$
2	1	0.5	$0.25(-0.5+\sqrt{4.25}) \approx 0.3904$
3	2	0	$0.25(1+\sqrt{17}) \approx 1.2808$

to decrease for all $\eta > 1/\sqrt{2} \approx 0.7071$. Certainly, the unit price will exceed the marginal cost for every $\eta > \eta(w) = 0.25(-1+w+\sqrt{w^2+6w+1})$, where $w = \theta_{\min} - c$.

Basically, examples of both scenarios exhibit a similar pattern of the variables described in Table 1. A possible interpretation of the pricing strategy is the following. For the owner of an amusement park such as Disneyland, it is optimal to charge a high admission fee and give the rides away as long as most customers are enthusiasts of the amusement park (that is, for sufficiently high η). Otherwise, it is optimal to charge a high price per ride, but entry is free. The latter case is referred to as the "razor-and-blades" pricing strategy in Schmalensee (2015, p.19), which is summarized as, "Give away the razor and make money on the blades".

Example 1. Set $\theta_{\max} = 2$ and $c = 0.5$. In this case, the optimal unit price and the optimal membership fee are the following:

$$\begin{cases} p(\eta) = \dfrac{\eta^2 + 2.5\eta + 0.5}{2\eta^2 + 2\eta + 1}, \\ q(\eta) = \dfrac{4.5\eta^4}{(2\eta^2 + 2\eta + 1)^2}. \end{cases}$$

Figure 4 shows a notable feature of the optimal unit price. The unit price increases with the first-order stochastic shift parameter η, when the value of η is sufficiently low.

Next, I shall examine the effect of a shift in η on the marginal consumer

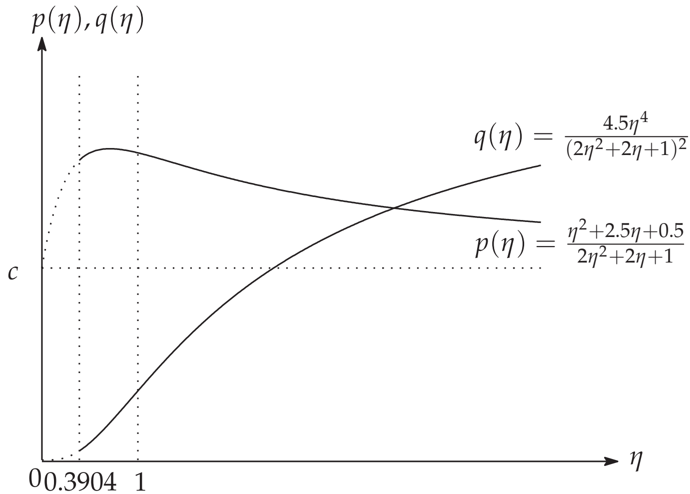

Figure 4　Optimal unit price and membership fee with $\theta_{\max} = 2$ and $c = 0.5$

type. The function $\theta(\eta)$ is strictly increasing and strictly concave in η:

$$\theta(\eta) = \frac{4\eta^2 + 2.5\eta + 0.5}{2\eta^2 + 2\eta + 1}.$$

In this case, the optimal marginal type is depicted in Figure 5. According to Figure 5, it is optimal for the firm to exclude low types.

It remains to examine whether this kind of pricing strategy is profitable or not. The maximized expected profit as a function of the parameter η is given by

$$\mathbb{E}\pi(\eta) = \left(\frac{1.5\eta^2}{\eta+1}\right)\left(\frac{1.5(\eta+1)}{2\eta^2+2\eta+1}\right)^{\frac{\eta+1}{\eta}}.$$

Figure 6 shows the graph of the maximized expected profit. It turns out that an increase in the first-order stochastic shift parameter η has a positive impact on profits in the relevant range.

The predicted values of the expressions summarized in Corollary 1 in this

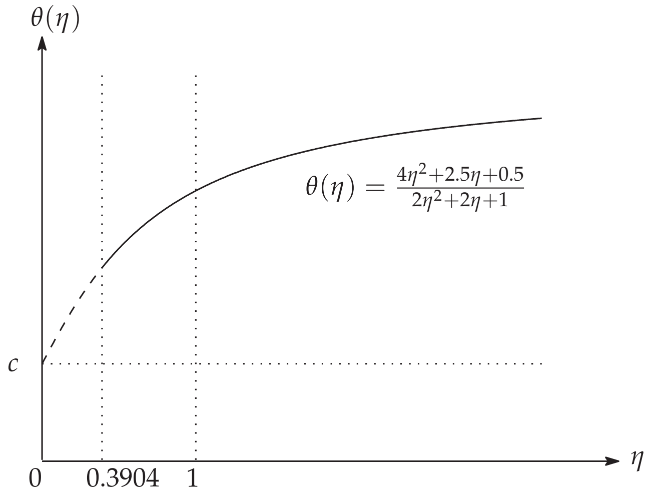

Figure 5 Optimal marginal type with $\theta_{\max} = 2$ and $c = 0.5$

case are given in Table 3.

Table 3 Comparative statics under $(\theta_{\max}, \theta_{\min}, c) = (2, 1, 0.5)$

	η						
	0.5	0.75	1	1.25	1.5	2	4
$p(\eta)$	0.8000	0.8104	0.8000	0.7830	0.7647	0.7308	0.6463
$q(\eta)$	0.0450	0.1084	0.1800	0.2503	0.3153	0.4260	0.6853
$\theta(\eta)$	1.1000	1.2759	1.4000	1.4906	1.5588	1.6539	1.8171
$Y(\eta)$	0.6000	0.7759	0.9000	0.9906	1.0588	1.1539	1.3171
$\tilde{\pi}(\eta)$	0.2250	0.3491	0.4500	0.5307	0.5956	0.6923	0.8780
$\mathbb{E}\pi(\eta)$	0.1823	0.2270	0.2700	0.3094	0.3452	0.4073	0.5742

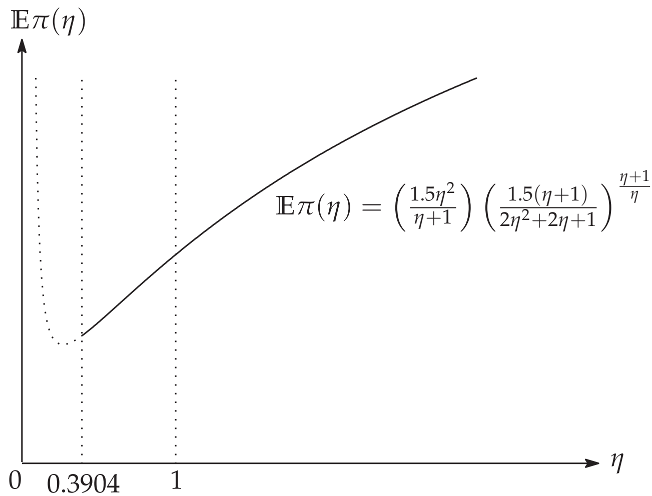

Figure 6 Maximized expected profit with $\theta_{\max} = 2$ and $c = 0.5$

Example 2. Set $\theta_{\max} = 3$ and $c = 0$. In this case, the optimal unit price and the optimal membership fee are the following.

$$\begin{cases} p(\eta) = \dfrac{3\eta}{2\eta^2 + 2\eta + 1}, \\ q(\eta) = \dfrac{18\eta^4}{(2\eta^2 + 2\eta + 1)^2}. \end{cases}$$

Figure 7 shows that the unit price always decreases with the first-order stochastic shift parameter η, whereas the membership fee increases.

Next, I shall examine the effect of a shift in η on the marginal consumer type. The function $\theta(\eta)$ is strictly increasing and strictly concave in η:

$$\theta(\eta) = \frac{3\eta(2\eta + 1)}{2\eta^2 + 2\eta + 1}.$$

In this case, the optimal marginal type is depicted in Figure 8:

Figure 9 shows the graph of the maximized expected profit. It tends to

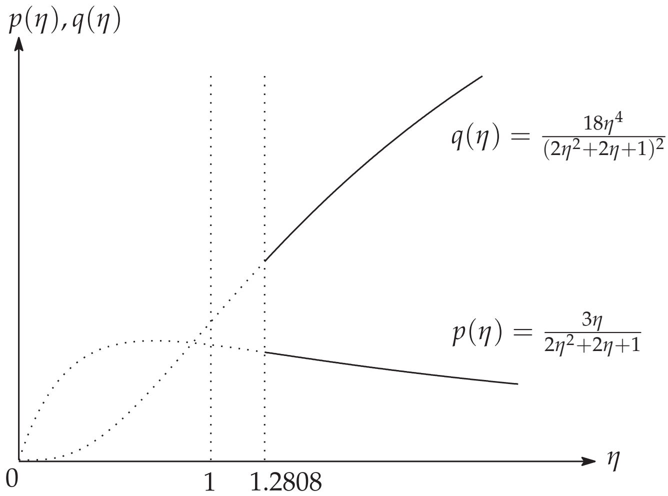

Figure 7 Optimal unit price and membership fee with $\theta_{\max} = 3$ and $c = 0$

be S-shaped in the parameter η.

$$\mathbb{E}\pi(\eta) = \left(\frac{3^{\frac{2\eta+1}{\eta}} \eta^2}{\eta + 1} \right) \left(\frac{\eta + 1}{2\eta^2 + 2\eta + 1} \right)^{\frac{\eta+1}{\eta}}.$$

Table 4 illustrates some specific results for the case $\theta_{\max} = 3$ and $c = 0$.

For instance, Leland and Meyer (1976) examine two-part tariff and two-block pricing when firms maximize profits facing heterogeneous consumer tastes. According to Leland and Meyer (1976, Theorem 2), a two-part tariff will yield strictly greater profits than any uniform pricing strategy. Due to such strict dominance ordering between uniform price and two-part tariff, it seems reasonable to focus on profit maximizing price schedule within two-part tariffs. The recent work by Iyengar and Gupta (2009) similarly analyze the impact of tariff structure on firm profitability within two-part and three-part tariffs.

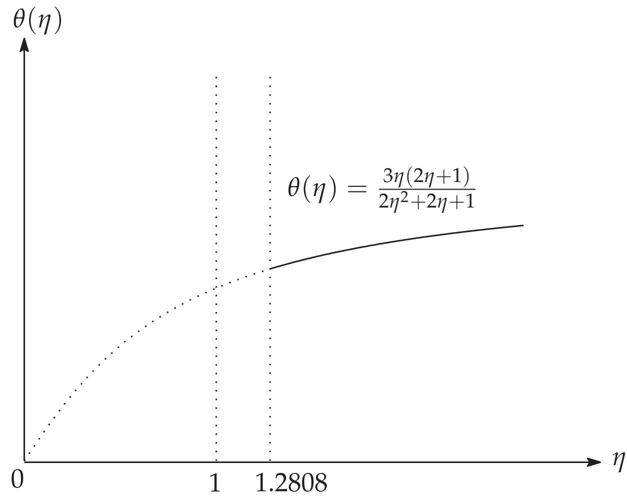

Figure 8　Optimal marginal type with $\theta_{\max} = 3$ and $c = 0$

5.　Conclusions

The paper has focused exclusively on the structure of the optimal two-part tariff schedule for the case of a profit-maximizing monopolist. The paper showed that the profit-maximizing monopolist sets his two-part tariff to equate the marginal rates of substitution in terms of the profit contribution by the average admitted consumers and the marginal rates of substitution of the marginal type. The notion of marginal rates of substitution is interpretable economically. A possible concern is whether similar conditions hold under the optimal multiblock tariff for future research.

The paper has analyzed the impact of the heterogeneity in consumer preferences. To highlight the principal effects of a shift in the distribution of the heterogeneity in consumer taste, I have developed a model under the assumption that demand curves are exactly parallel. Explicit model-

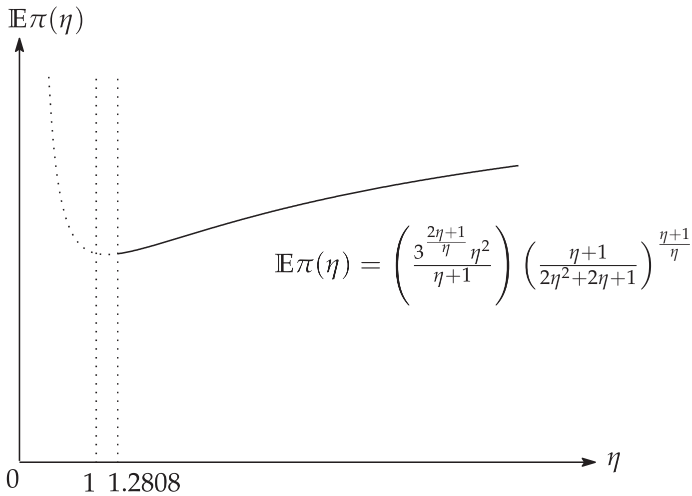

Figure 9 Maximized expected profit with $\theta_{\max} = 3$ and $c = 0$

ing of the firm's profit maximization problem benefits. It was possible to solve for not only the optimal two-part tariff schedule but also the analytical expression for the maximized expected profit for the firm implied by heterogeneity in consumer's valuation.

The model was special in at least two respects. While the assumptions of linear demands and constant unit costs of production are at least familiar, the assumption that demand curves are exactly parallel was particularly strong. As a general matter it is desirable to study more general demand structures.

Finally, throughout the paper I have focused on the profit maximization problem, but two-part tariffs have been applied to problems of social welfare maximization as well. Future research could examine the properties of the welfare maximizing pricing strategies.

Table 4 Comparative statics under $(\theta_{\max}, \theta_{\min}, c) = (3, 2, 0)$

	η					
	1.29	1.5	1.75	2	4	10
$p(\eta)$	0.5602	0.5294	0.4941	0.4615	0.2927	0.1357
$q(\eta)$	1.0445	1.2613	1.4954	1.7041	2.7412	3.6854
$\theta(\eta)$	2.0055	2.1177	2.2235	2.3077	2.6342	2.8507
$Y(\eta)$	2.0055	2.1177	2.2235	2.3077	2.6342	2.8507
$\tilde{\pi}(\eta)$	2.1680	2.3824	2.5941	2.7692	3.5122	4.0724
$\mathbb{E}\pi(\eta)$	2.1587	2.1916	2.2449	2.3041	2.7315	3.3672

Acknowledgments

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